# Least squares optimal realisation of autonomous LTI systems is an eigenvalue problem[1]

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We outline the solution of a long-standing open problem in *system identification and signal processing*, on how to find the *best least squares realisation of an autonomous linear time-invariant (LTI) dynamical system from given data*.

The global optimum is found among all stationary points of a least squares objective function, which we show to correspond to the eigen-tuples of a multi-parameter eigenvalue problem (MEVP). Such an MEVP can be solved by applying Forward (multi-) Shift Recursions to the given set of multivariate polynomial equations, generating so-called block Macaulay matrices, the null space of which can be modelled as the observability matrix of a multi-dimensional shift-invariant linear commutative singular system. The state equations of this system can be found from multi-dimensional realisation theory. From the corresponding eigen-tuples, one can then find the optimal parameters of the best LTI autonomous model.

Our solution methodology uses ingredients from algebraic geometry, operator theory, multi-dimensional system theory and numerical linear algebra, and ultimately requires as basic building blocks only the singular value decomposition and eigen-solvers.

Surprisingly enough, the conclusion is that the globally optimal model in 1D least squares realisation, can be found exactly from multi-dimensional realisation. In addition, we describe several new, previously unknown, properties that characterise the optimal model and its behaviour.

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